Constraints on Couplings
of Extra Gauge Bosons from $e^+e^- \to \nu\bar{\nu}\gamma$

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Abstract

If extra gauge bosons are observed at the LHC or at a linear e^+e^- collider, the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ can give information on the couplings $W'\nu l$ and $Z'\nu\bar{\nu}$. The total cross section and polarization asymmetries are the most sensitive observables for such a measurement. Small systematic errors are crucial to obtain reasonable coupling constraints, especially for a high luminosity collider.

1   Introduction

Extra charged (W') and extra neutral (Z') gauge bosons are predicted by many extensions of the Standard Model (SM). The masses and the couplings of the extra gauge bosons to fermions depend on the free parameters of the theory extending the Standard Model (SM). They must be determined by experiment. Hence, the observation of extra gauge bosons would teach us an important lesson on physics beyond the SM. Therefore, the search for these particles is included in the research program of every future collider.

Recent collider data are consistent with the SM. In particular, they suggest that extra gauge bosons predicted in Grand Unified Theories (GUT's) must be heavier than ∼500 GeV [1]. We therefore expect that extra gauge bosons would manifest themselves as (small) deviations of some observables from the SM predictions at future linear e^+e^- colliders. At LHC, they would show up through peaks in the invariant mass distribution of their decay products. Two cases are possible. If the SM is confirmed at future colliders, impressive improvements on the exclusion limits on extra gauge bosons can be achieved, see e.g. [2, 3] for a review and further references. For extra gauge bosons with masses considerably smaller than their search limit in the same reaction, some information on their masses and couplings can be obtained. See again [2, 3] for a review and further references.

In most models, the reactions $e^+e^- \to (\gamma, Z, Z') \to f\bar{f}$ and $pp \to Z'(W')X$ are most sensitive to a Z' or W'. We suppose here that a signal of extra gauge bosons is detected in one of these reactions and its mass and some of its couplings are measured.

In earlier contributions [4], we investigated the potential of the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ in obtaining search limits for a W' appearing in different models. For some models, these limits can compete with those from the LHC. In contrast to the limits from hadron colliders, the limits from e^+e^- collisions are independent of unknown details of structure functions and quark mixing. This is a good process at an e^+e^- collider for putting bounds on a W'.

In this contribution, we show where the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ can contribute complementary information to the coupling measurements of extra gauge bosons. A more detailed paper is forthcoming.

2   Assumptions and Calculation

We show constraints on the couplings $L_\nu(Z')$, $R_\nu(Z')$ of extra neutral gauge bosons to neutrinos and on the couplings $L_l(W')$, $R_l(W')$ of extra charged gauge bosons to leptons. The couplings $L_f(V)$ and $R_f(V)$ of the left- and right-handed fermions $f_{L,R}$ to the vector boson V are defined by the following interaction lagrangian,

$$\mathcal{L} = V_\mu \bar{f}_L \gamma^\mu f_L \cdot L_f(V) + V_\mu \bar{f}_R \gamma^\mu f_R \cdot R_f(V). \tag{1}$$

Two-dimensional constraints (95% confidence) correspond to

$$\chi^2 = \sum_i \left(\frac{O_i(SM) - O_i(SM + Z' + W')}{\Delta O_i} \right)^2 = 5.99. \tag{2}$$

In equation (2), $O_i(SM)$ is the expectation for the observable O_i by the SM, $O_i(SM+Z'+W')$ is the prediction of the extension of the SM and ΔO_i is the expected experimental error. The index i numbers different observables as, for example, the total cross section $\sigma_T = \sigma$ and A_{LR}.

The physical input for our numerics is $M_W = 80.33\,GeV$, $M_Z = 91.187\,GeV$, $\sin^2\theta_W = 0.23124$, $\alpha = 1/128$ and $\Gamma_Z = 2.49\,GeV$.

Let E_γ and θ_γ denote the photon's energy and angle in the e^+e^- center-of-mass, respectively. We have restricted the range of E_γ and θ_γ as follows:

$$10\,GeV < E_\gamma < \frac{\sqrt{s}}{2}(1 - M_Z^2/s) - 6\Gamma_Z, \tag{3}$$

$$10^o < \theta_\gamma < 170^o, \tag{4}$$

so that the photon may be detected cleanly. The upper bound on E_γ ensures that the photons from the radiative return to the SM Z resonance are rejected. As well, the angular cut eliminates the collinear singularity arising when the photon is emitted parallel to the beam.

The most dangerous background to the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ comes from radiative Bhabha scattering. It can be eliminated by a cut on the transverse momentum of the photon,

$$P_{T,\gamma} > \sqrt{s}\sin\theta_\gamma \sin\theta_v/(\sin\theta_\gamma + \sin\theta_v), \tag{5}$$

where θ_v is the minimum angle for veto detectors. We take $\theta_v = 25\,\text{mrad}$.

The unpolarized cross section is composed of polarized cross sections as $\sigma = \sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR} = \sigma_L + \sigma_R$. Note that $\sigma_{LL} = \sigma_{RR} \neq 0$ only in models where the W' has non-zero couplings to left- and right-handed fermions simultaneously. Even in this case, we have $\sigma_{LL}, \sigma_{RR} \ll [\sigma(SM) - \sigma(SM + Z' + W')]$ due to present experimental bounds on extra gauge bosons and because σ_{LL} and σ_{RR} do not interfere with the SM.

We consider polarization asymmetries involving polarized electrons only, $P^- = 90\%$, $A_{LR} = (\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$, and polarization asymmetries involving both polarized beams, $P^- = 90\%$, $P^+ = 60\%$, $A_{LR} = (\sigma_{LR} - \sigma_{RL})/(\sigma_{LR} + \sigma_{RL})$ and devote half of the luminosity to each polarization combination. The case of two polarized beams corresponds to an effective

polarization $P_{eff} = (P^- + P^+)/(1 + P^- P^+) = 97.4\%$ [5] of the electron beam. Note that the use of the effective polarization is exact for $\sigma_{LL} = \sigma_{RR} = 0$. If this is not the case, it remains a good approximation.

We assume systematic errors of 1% for both observables σ and A_{LR}, although we know that this assumption is very optimistic for σ.

We have performed two independent calculations. In the first calculation, the squared matrix element was obtained in the conventional trace formalism. The phase space of the neutrinos was integrated out analytically. This results in (rather long) analytical formulas for the doubly differential cross section $d^2\sigma/(dE_\gamma d\cos\theta_\gamma)$. All analytical integrals were checked numerically to a precision of at least 10 digits. Artificial singularities in the physical phase space appeared from the partial fraction decomposition of the first integration. These singularities were proven to cancel analytically after the second integration and an appropriate Taylor expansion. All algebraic manipulations were carried out with the algebraic manipulation program `form` [6]. The remaining one (two) integrations to obtain simple distributions (total cross sections) were done using an adaptive Simpson integration routine.

In a second calculation, the helicity amplitudes were calculated by spinor techniques. They were squared numerically and summed over the helicities of the final states. The result was integrated using Monte Carlo techniques.

The total cross sections from both calculations agree within the Monte Carlo errors. For the SM, they agree with the predictions of `COMPHEP` [7].

3 Extra Neutral Gauge Bosons

In figures 1 and 2, we assume that there is a signal from a Z' and no signal from a W'. This can happen in models where the W' has purely right-handed couplings and the right-handed neutrino is heavy. If there is a signal for a W', a similar analysis could be done including the known W' parameters. The experimental errors of these parameters would then enlarge the errors of the measurements shown in the next two figures. However, it would leave our main conclusions unchanged.

We find that the process $e^+ e^- \to \nu\bar{\nu}\gamma$ can give some constraints to the couplings of the Z' to SM neutrinos *below* the Z' resonance.

In figures 1 and 2, we assume a heavy repetition of the SM Z boson. We see that we can get some constraints even in the case where the Z' is considerably heavier than the center-of-mass energy. The region which cannot be resolved by the observables is between the two lines and contains the couplings of the assumed model. This choice also shows our normalization of the couplings. For the cases where only one bounding line is shown, the second line is outside the figure. $R_\nu(Z')$ and $L_\nu(Z')$ are mainly constrained by the interference of the Z' contributions with the SM contributions. Mainly the Z' coupling to left-handed neutrinos can be constrained. This makes the constraints especially simple.

Figure 1 illustrates the effect of different observables and different experimental parameters on these constraints. The total cross section gives the strongest constraint. The constraint from A_{LR} is shown for one and two polarized beams. The constraints from energy and angular distributions with 10 equal size bins give no improvement. The constraint for a low luminosity linear collider is also shown in figure 1. A systematic error of 1% relaxes the constraints considerably and dilutes the advantage of high luminosity. We see that small systematic errors and a high luminosity collider are highly desired for the proposed model measurement.
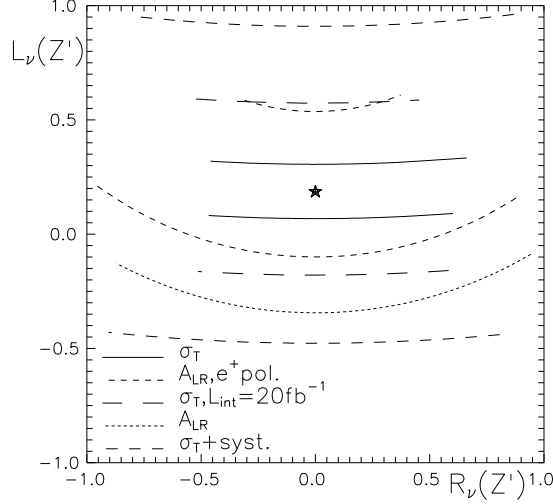
Figure 1 *Constraints on the $Z'\nu\bar\nu$ couplings $R_\nu(Z')$ and $L_\nu(Z')$ by different observables, see text. $\sqrt{s} = 0.5\,TeV$, $M_{Z'} = 1.5\,TeV$. $L_{int} = 200\,fb^{-1}$ except the indicated case were it is $20\,fb^{-1}$. The polarization of the electron beam is 90%. The polarization of the positron beam (where indicated) is 60%. Where indicated, the systematic error is 1% for σ and A_{LR}, otherwise it is zero . The assumed model is a Z' in the Sequential Standard Model (SSM), indicated by a star.*
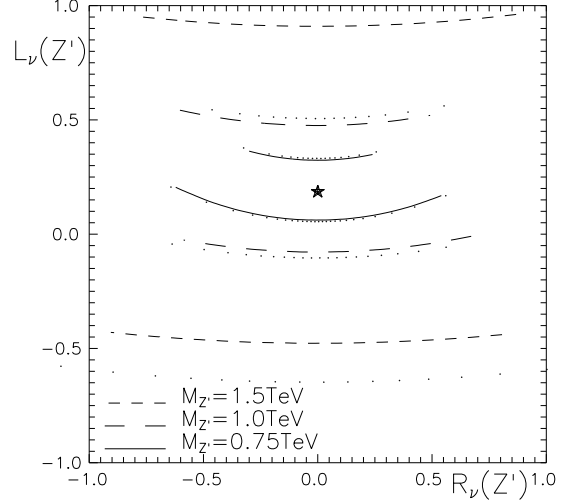
Figure 2 *Constraints on $R_\nu(Z')$ and $L_\nu(Z')$ below the Z' peak by σ only. The lines show the results for three different Z' masses. The dots indicate how the constraints relax if the error of the $Z'e^+e^-$ coupling measurement is included. $\sqrt{s} = 0.5\,TeV$, $L_{int} = 200\,fb^{-1}$ A systematic error of 1% is included. The assumed model (SSM Z') is indicated by a star.*

Figure 2 shows the possible constraints on $R_\nu(Z')$ and $L_\nu(Z')$ including a systematic error of 1% for three representative Z' masses. The constraints become much stronger for smaller Z' masses. So far, we assumed that the $Z'e^+e^-$ couplings $R_e(Z')$ and $L_e(Z')$ are precisely known. However, they must be measured (with errors) by experiment. Figure 4b of reference [8] illustrates such a measurement for a collider with conventional luminosity. The errors from reference [8] are taken to estimate its influence on the $R_\nu(Z')$, $L_\nu(Z')$ constraint. Our input for the errors of the $Z'e^+e^-$ couplings for $M_{Z'} = 1.0\,TeV$ and $0.75\,TeV$ are obtained from those for $1.5\,TeV$ by the scaling relation (2.63) in [2]. We see that the uncertain knowledge of the $Z'e^+e^-$ couplings leads to weaker constraints on $R_\nu(Z')$ and $L_\nu(Z')$. However, figure 2 shows that this effect is only important for a very heavy Z' where its couplings are already weakly constrained. The influence of the errors of the $Z'e^+e^-$ coupling measurement is negligible for a relatively light Z'.

Finally, we mention that there is no sign ambiguity in the measurement of $R_\nu(Z')$ and $L_\nu(Z')$, if the signs of the $Z'e^+e^-$ couplings are known. Remember [8] that the $Z'e^+e^-$ couplings have a two-fold sign ambiguity if measured in the reaction $e^+e^- \to f\bar{f}$ alone. If the sign ambiguity in the $Z'e^+e^-$ couplings is removed [2], i.e. by measurements of the reaction $e^+e^- \to W^+W^-$ below the Z' resonance or by measurements at the Z' resonance, it disappears also in the proposed constraint of $R_\nu(Z')$ and $L_\nu(Z')$.

4 Extra Charged Gauge Bosons

In figures 3 to 6, we assume that there is no signal from a Z' but a signal from a W'. This can happen in models where the W' is considerably lighter than the Z'. We recognize that this particular scenario is unlikely in the context of the models we consider. For instance, in the UUM model, the W and Z masses are approximately equal and there would most likely be a signal observed for the Z' in addition to the W'. So it should be understood that our results for the case of a W' only represent an estimate of the reach of this process in constraining W' couplings, rather than precision limits in the context of a fuller understanding of the physics realized in nature. We use this simple scenario in order to indicate sensitivity to various parameters, such as the observables used and the luminosity. In the case of a Z' signal, the knowledge about this particle should be included in the following analysis. Again, the experimental errors of the measurements of the Z' parameters would enlarge the errors of the W' measurements but not change our main conclusions.

We find that the process $e^+e^- \to \nu\bar{\nu}\gamma$ can give constraints on the couplings $L_l(W')$ and $R_l(W')$ for W' masses considerably larger than the center-of-mass energy.
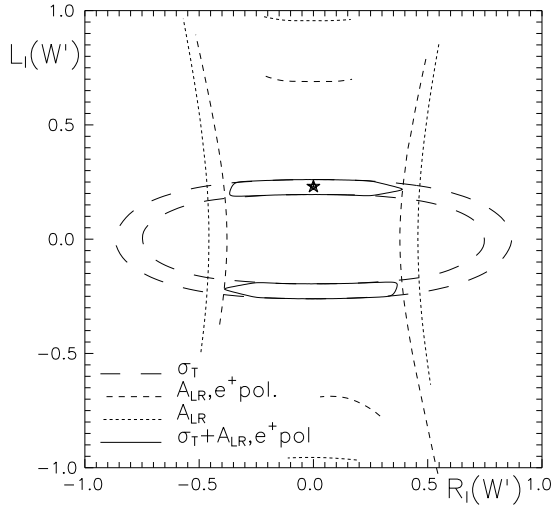
Figure 3 *Constraints on the W' couplings by σ and A_{LR}. $\sqrt{s} = 0.5\,TeV$, $L_{int} = 200\,fb^{-1}$ and $M_{W'} = 1.5\,TeV$. The constraint by σ and A_{LR} combined is also shown. Only statistical errors are included in this figure. 90% electron and, where indicated, 60% positron polarization are assumed. The assumed model (SSM W') is indicated by a star.*
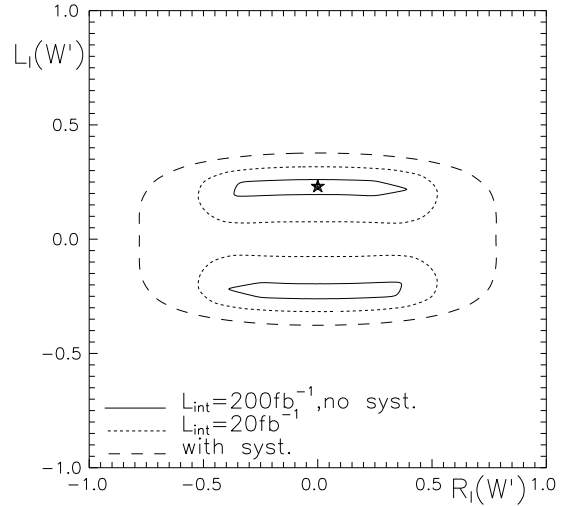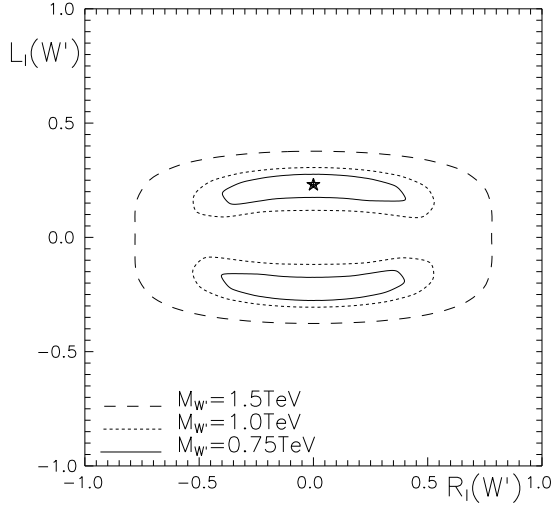
Figure 4 *Constraints on the W' couplings by σ and A_{LR} combined. 90% electron and 60% positron polarization are assumed. $\sqrt{s} = 0.5\,TeV$, $M_{W'} = 1.5\,TeV$. $L_{int} = 200\,fb^{-1}$ except the indicated case where it is $20\,fb^{-1}$. Only statistical errors are included except the indicated case where a systematic error of 1% is included for σ and A_{LR}. The assumed model (SSM W') is indicated by a star.*

Figure 3 is similar to figure 1 but it shows the constraint on the W' couplings. For illustration, we assume that there exists a W' with couplings as the SM W but with a mass of $1.5\,TeV$. We get some constraints on the couplings even in this case. The constraints

from energy and angular distributions give no improvement for the considered model. The constraint from A_{LR} is complementary to that from the total cross section. It is shown for one and two polarized beams. It turns out that σ and A_{LR} together give the best constraint on the coupling. The constraints on the W' couplings always have a two-fold sign ambiguity, i.e. nothing is changed by a simultaneous change of the sign of $L_l(W')$ and $R_l(W')$. This ambiguity is obvious from the amplitude where W' couplings always enter in products of pairs.

In figure 4, we show constraints on the W' couplings from σ and A_{LR} combined. We have two well separated regions for high luminosity and no systematic error (compare figure 3). These two regions come closer together for low luminosity and no systematic error. We are left with one large region after the inclusion of a systematic error of 1%. As in the case of extra neutral gauge bosons, a small systematic error *and* a high luminosity are necessary for a coupling measurement.

Figure 5 *Constraints on the W' couplings by σ and A_{LR} combined for different W' masses. 90% electron and 60% positron polarization, $\sqrt{s} = 0.5\,TeV$ and $L_{int} = 200\,fb^{-1}$ are assumed. A systematic error of 1% is included for σ and A_{LR}. The assumed model (SSM W') is indicated by a star.*
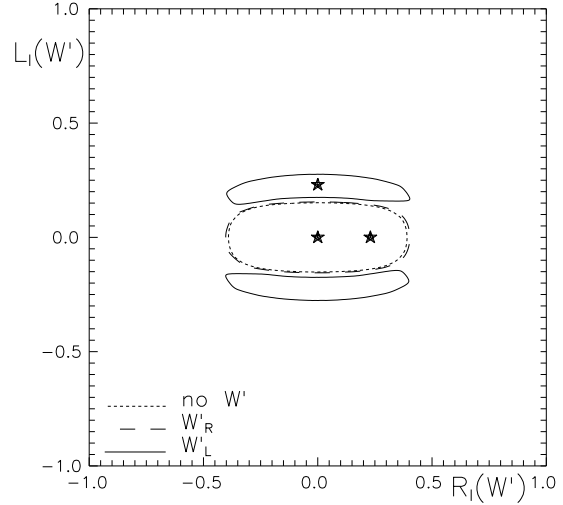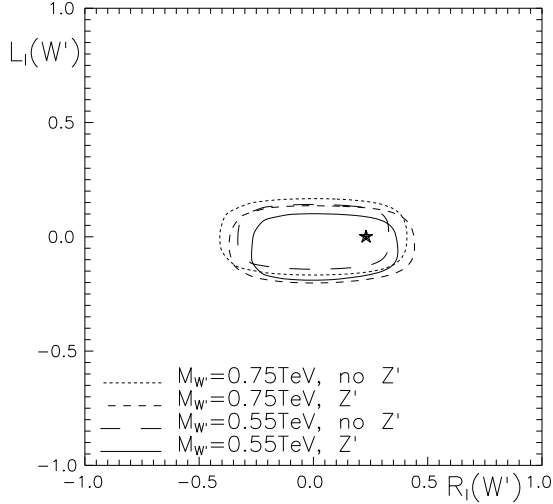
Figure 6 *Constraints on the W' couplings by σ and A_{LR} combined for different W' scenarios. 90% electron and 60% positron polarization, $\sqrt{s} = 0.5\,TeV$, $L_{int} = 200\,fb^{-1}$ and $M_{W'} = 0.75\,TeV$ are assumed. A systematic error of 1% is included for σ and A_{LR}. The assumed models are indicated by a star.*

In figure 5, we show how the constraints on the W' couplings vary for different W' masses. The constraint for $M_{W'} = 1.5\,TeV$ is identical to that shown in figure 4. We see that the constraint on the W' masses improves dramatically for lower W' masses.

Figure 6 illustrates the possible discrimination between different models. We see that a W' with SM couplings (W'_L) can be separated from the SM. A W' with pure right-handed couplings (W'_R) with a strength of the left-handed coupling of the SM W cannot be distinguished from the SM case.

From the amplitude of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, it can be seen that the constraints shown in figs. 3 to 6 are, to good approximation, valid for the combinations $L_l(W')/M_{W'}$ and

$R_l(W')/M_{W'}$, and not for the couplings and the mass separately. We have fixed so far the W' mass for illustration. If a W' is found with a mass different from our assumptions, the constraint on its couplings can then be obtained by the appropriate scaling of our results as long as $M_{W'} \gg \sqrt{s}$.

Figure 7 *Constraints on the W' couplings by σ and A_{LR} combined in the LR model with $\rho = 1$ and $\kappa = 1$ for different W' masses and different fitting strategies (see text). 90% electron and 60% positron polarization, $\sqrt{s} = 0.5\,TeV$ and $L_{int} = 200\,fb^{-1}$ are assumed. A systematic error of 1% is included for σ and A_{LR}. The assumed model is indicated by a star.*
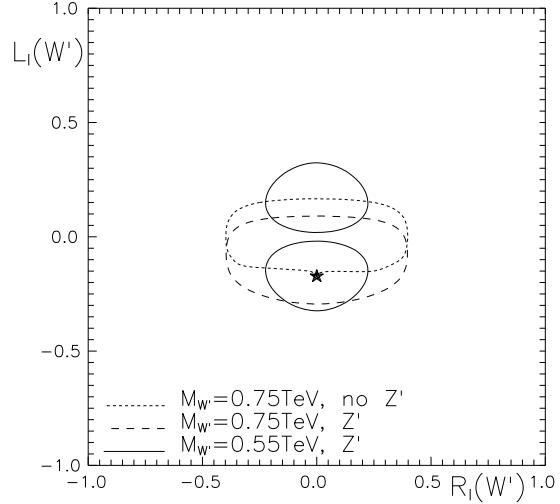
Figure 8 *Constraints on the W' couplings by σ and A_{LR} combined in the UUM with $\sin\psi = 0.6$ for different W' masses and different fitting strategies (see text). 90% electron and 60% positron polarization, $\sqrt{s} = 0.5\,TeV$ and $L_{int} = 200\,fb^{-1}$ are assumed. A systematic error of 1% is included for σ and A_{LR}. The assumed model is indicated by a star.*

We considered model independent bounds on the couplings of extra gauge bosons neglecting the existence of other extra gauge bosons, i.e. of the W' in the case of Z' constraints or of the Z' in the case of W' constraints. However, in the general case, extra neutral and extra charged gauge bosons simultaneously influence the observables.

In figure 7, we assume that the left-right-symmetric model is true. See reference [4] for conventions. In the first case, we take $M_{W'} = 0.75\,TeV$. It follows that $M_{Z'} = 0.90(1.27)\,TeV$ for $\kappa = 1$ and $\rho = 1(2)$. We show the constraints on the couplings of the W' for $\rho = 1$ obtained by two different fitting strategies: first ignoring the Z' completely, and second taking the Z' into account exactly without any errors. We see that the two curves are quite close. The reason is that the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ is not very sensitive to such a Z'.

The more realistic case that the Z' parameters are measured with some errors would produce a constraint between the two curves of figure 7. The case $\rho = 2$ predicts a heavier Z'. This would produce two curves, which differ even less from each other than those for $\rho = 1$. To demonstrate how the constraints change for a larger signal, we repeated the same procedure with $M_{W'} = 550\,GeV$. This number (and the mass of the associated Z') are at the edge of the present exclusion limit [1]. However, the constraints get only a minor improvement.

Figure 8 is similar to figure 7, but now we assume that the Un-Unified model is true. See again reference [4] for conventions. We consider two different mass scenarios, $M_{W'} = M_{Z'} = 0.75\,TeV$ and $M_{W'} = M_{Z'} = 0.55\,TeV$. We show the constraints on the couplings of the W' obtained by the same two fitting strategies as demonstrated in figure 7. Already for masses of $0.75\,TeV$, the two curves differ much more than was the case for the LR model. For masses of $0.55\,TeV$, the wrong fitting strategy gives always a χ^2 above 15, i.e. no allowed region in our case. This shows that such a light Z' cannot be ignored in the fitting procedure.

Other reactions as $e^+e^- \to f\bar{f}$ or hadron collisions are more sensitive to a Z' than $e^+e^- \to \nu\bar{\nu}\gamma$. In almost all cases, they will detect a Z' signal in the cases where the Z' contribution is relevant for a W' constraint by $e^+e^- \to \nu\bar{\nu}\gamma$. We conclude from figures 7 and 8 that this information from other experiments is needed for a reliable constraint on the W' coupling.

5 Conclusions

We found that the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ can give interesting constraints on the couplings of extra gauge bosons to leptons. The total cross section and polarization asymmetries are the most sensitive observables for this measurement. The assumed universal systematic error of 1% seems to be very optimistic for the total cross section. Already this small systematic error almost dilutes the advantages gained from the high luminosity option of the collider. A polarized electron beam is crucial. A polarized positron beam is not essential but gives a quantitative improvement of the coupling constraints.

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